Math 213: Calculus III – Exam 3

April 22, 2026

Calculators, cell phones, and formula sheets are **NOT** permitted. Show all work clearly.

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1. (10 pts): Consider the region R in the first quadrant bounded by the curves $y = x^2$ and $y = 5x$. Sketch the region R. Set up and evaluate a double integral to find the area of R.

2. (12 pts): Evaluate the integral by converting to polar coordinates: where R is the region in the upper half-plane ($y \geq 0$) bounded by the circle $x^2 + y^2 = 9$ and the x axis.

$$\iint_R (x^2 + y^2)\ dA$$

3. (12 pts): Set up, but DO NOT EVALUATE, a triple integral using cylindrical coordinates for the mass of the solid region bounded below by the xy-plane, on the sides by the cylinder $x^2 + y^2 = 4$, and above by the sphere $x^2 + y^2 + z^2 = 16$ where the density at each point is given by $\rho(x, y, z) = e^x$

4. (12 pts) Set up a triple integral in spherical coordinates for the $\iint\int z dV$ on the region bounded below by the cone $z^2 = x^2 + y^2$ and on top by the sphere $x^2 + y^2 + z^2 = 9$.

5. (16 pts): Show that $\vec{F}(x, y, z) = (2xy)\ \mathbf{i} + (x^2 + z)\ \mathbf{j} + (y)\ \mathbf{k}$ is a conservative vector field. Then find a potential function $f(x,\ y,\ z)$ such that $\nabla f = \vec{F}$. Finally, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is any smooth path from $(0,\ 0,\ 0)$ to $(2,\ -1,\ 3)$.

6. (12 pts): Evaluate the path integral $\int_C (x + \dfrac{1}{y+1})\ ds$ along the line segment from (0,0) to (3,2) .

7. (12 pts): Use Green's Theorem to find the counterclockwise circulation of the vector field $\vec{F}(x, y) = (x - y^2)\ \mathbf{i} + (x^2 - 2y)\ \mathbf{j}$ around the boundary of the unit square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$. Then use Green’s Theorem to find the outward flux.

8. (14 pts) Use the transformation $u = xy$ and $v = \dfrac{y}{x}$ to convert $\iint e^{xy} dxdy$ over the region bounded by $y = \dfrac{1}{x}; y = \dfrac{4}{x}; y = x$ and $y = 5x$ into a double integral in u and v. First find x and y in terms of u and v. Then calculate the Jacobian and finally set up the integral. DO NOT EVALUATE.